Drawing and Visualisation Research

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tracey@lboro.ac.uk

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GEOMETRIC REASONING AND DRAWING: POSSIBLE INTERCONNECTIONS AMONG STEM SUBJECTS AND ART

Lynn T. Goldsmith[a], Seymour Simmons[b], Ellen Winner[c], Lois Hetland[d], Craig D. Hoyle[e], Candace Brooks[f]

a Education Development Center, Inc., LGoldsmith@edc.org
b Winthrop University, simmonss@winthrop.edu
c Boston College, ellen.winner@bc.edu
d Massachusetts College of Art and Design lhetland@massart.edu
e Education Development Center, Inc., CHoyle@edc.org
f Education Development Center, Inc., CBrooks@edc.org

Cezanne's famous maxim, "interpret nature in terms of the cylinder, the sphere, the cone," (Loran, 2006: 8) describes a form of artistic envisioning that was of immense value for post-impressionist landscape painting, but that had origins in far earlier traditions in both western and non-western art. The kind of visual thinking Cezanne refers to is also important in the full range of "STEM" subjects—science, technology, engineering, and mathematics—where the ability to imagine and manipulate forms in space plays a fundamental role in problem solving (Cunningham, 2005; Hegarty 2010; Hegarty and Kozhevnikov, 1999). Here we describe a study that explores possible connections between artistic envisioning (Hetland, Winner, Veenema, and Sheridan, 2007) and geometry, an area of mathematics that relies heavily on spatial reasoning. We also consider possible applications of this study to drawing instruction. We set the stage with a short discussion of how visualising and drawing—both fundamental tools of the artist, designer, illustrator, and architect—can also be critical to thinking in STEM disciplines.

We first consider ways that mathematical thinking might influence the visual arts (and vice versa—how the visual arts might influence mathematical and scientific thinking). In terms of the former, the use of geometric shapes, forms, and patterns has a long and rich history in art from many cultures. An obvious example is Cubism, the Modernist movement that was inspired in equal measure by Cezanne's interpretation of nature and the geometrically stylised art of Africa and Oceania. Innovative for its time, Cubism was actually a relatively late manifestation of an artistic, scientific, and philosophical tradition that extends back at least to ancient Greece and the doctrine, in the words of historian of mathematics Morris Kline "that the essence of nature is mathematical law" (Kline, 1957: 623).

Greek architecture, for example, is based on the Golden Section; a similar attention to "ideal" proportions characterised the work of Renaissance artists and architects, where it found expression in such famous images as da Vinci's "Vitruvian Man.' The notion that nature was grounded in mathematical law inspired another well-known Renaissance innovation, linear perspective. Renaissance painters, "imbued with this belief [about the relationship between mathematics and nature]…struggled for over a hundred years to find a mathematical scheme which would enable them to depict the three-dimensional real world on a two-dimensional canvas. . . .It was very fortunate that the painters were also architects and engineers and, in fact, the best mathematicians of the fifteenth century" (Kline, 1957: 623).

What began as a solution to an artistic need became, over time, its own branch of mathematics: two hundred years after perspective was invented, "[p]rofessional mathematicians took over the investigations of these questions and developed a geometry of great generality and power. Its name is projective geometry" (Kline, 1957: 624). The interconnectedness between visual arts and STEM subjects is further illustrated by Kline's observation that Gérard Desargues, a self-taught engineer and architect, "sought to combine the many theorems on perspective [from projective geometry] in[to] compact form that would be useful to artists, engineers and stonecutters. . . .[Desargues' theorem is] still fundamental in the subject of projective geometry" (Kline, 1957: 626).

In the sciences the importance of visual thinking (and of the role of drawing and sketching as a way of representing ideas) is underscored in accounts of mental processes by figures such as Einstein (Hadamard, 1945), Watson and Crick (Watson, 1968), Kekulé (Rocke, 2010), and Tesla (McKim, 1972). In many such cases, ideas that take shape in part through visualisations may then become refined and communicated through sketches and diagrams. For example, the chemist Kekulé, who is famously reported as having identified the ring-like structure of the benzene molecule by interpreting a dream of a snake biting its own tail, recounted a similar instance of envisioning which led him "from valence theory to the much more consequential theory of chemical structure." After having had a vision while riding half-asleep on board an omnibus, he spent part of the night "committing at least sketches of these dream figures to paper. This was the birth of the structure theory" (Rocke, 2011: 63). McKim also notes that drawing played an important role in James Watson's thinking, noting that Watson recollected that an important idea regarding the structure of DNA "came while I was drawing the fused rings of adenine on paper" (McKim, 1972: 9).

For the visual artist and the STEM practitioner alike, drawing, particularly sketching, offers a means of representing structure and relationships, and for holding these still for inspection, reflection, and correction.

> *Drawing not only helps to bring vague inner images into focus. It also provides a record of the advancing thought stream. Further, drawing provides a function that memory cannot: the most brilliant imaginer cannot compare a number of images, side by side in memory, as one can compare a wall of tacked up idea-sketches (McKim, 1972: 10).*

McKim (1972) calls this use of drawing *graphic ideation*, as contrasted to drawing intended to communicate more fully formed ideas to others*.* While McKim describes drawing as an *aid to* thinking or *a reflection* of thought, however, Kantrowitz (2012) has argued that drawing *is* thinking. Indeed, her position reinforces McKim's suggestion that training in certain approaches to drawing—for example, rapid sketching—is actually training in the kinds of rich, pattern-seeking thought that can yield new and creative ideas. Although McKim suggests that other approaches to drawing such as careful rendering may impede creative thought, we argue here that the close analysis of shape and space required by observational or technical drawing and drafting may pay large dividends in terms of promoting STEM thinking, whether creative, critical, investigative, or reflective. In the remainder of this paper, such questions are considered as we discuss two research studies and an exploratory foray, each investigating possible relationships between training in the visual arts and geometric reasoning.

MIGHT VISUAL ARTS TRAINING IMPROVE GEOMETRIC REASONING? THREE EXPLORATORY STUDIES

The question of whether study of the visual arts might affect mathematical (specifically, geometric) thinking grew out of a meta-analysis of the research on connections between the arts and academic achievement (Winner and Hetland, 2000). Winner and Hetland concluded that there was little empirical evidence to support the frequently advanced claim that "the arts make you smarter" (i.e., that arts training results in improved academic achievement). They also concluded that the lack of evidence did not necessarily mean that there was no relationship, but rather that many of the studies were weak methodologically. Perhaps the most significant deficiency was a lack of articulation of the kinds of knowledge or skills developed in "parent" art domains that might reasonably be expected to transfer to other academic domains. Their subsequent ethnographic study of art instruction in two arts-based high schools was an effort to identify such skills. Hetland and colleagues identified eight "studio habits of mind" that characterised students' training across the five studio classes they studied (Hetland, Winner, Veenema, and Sheriden, 2007, 2013).

One of the eight habits they identified, "envisioning," seemed the most promising for exploring transfer from visual arts to STEM domains. Hetland et al. (2007, 2013) described envisioning as follows.

> *Envisioning includes the acts of generating mental images so that one can imagine how a work will look, and planning ways to achieve that image…When we envision, we imagine and generate images of possibilities in our mind…The translation from model to representation cannot be done without envisioning. Artists aim to represent not only the surface aspects of their models but also the underlying structure or geometry—for example, the axis of the head versus the axis of the body, the torso as a trapezoid, the triangular relation between two figures (Hetland et al., 2007: 48).*

As suggested earlier, virtually every STEM discipline calls on visual or spatial thinking—thinking that likely shares characteristics with artistic envisioning. For example, geometer Walter Whiteley observed:

> *I am a research mathematician, working in discrete applied geometry. My own practice of mathematics is deeply visual: the problems I pose, the methods I use;*

the ways I find solutions; the way I communicate my results. The visual is central to mathematics as I experience it (Whiteley, 2004: 1).

With examples such as these in mind, we have undertaken two studies designed to explore connections between training in the visual arts (in which envisioning constitutes a significant component) and students' ability to reason geometrically. In both cases, we found that visual arts students outperformed non-art student peers on geometric reasoning tasks.

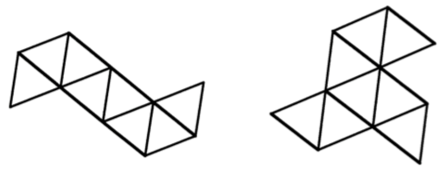
Study 1: Geometric Reasoning in Collegiate Studio Arts Majors and Psychology Majors

In this study we explored the question of whether studio arts majors (who it is assumed were trained to engage in artistic envisioning) would demonstrate stronger geometric reasoning than psychology majors, whose content area major did not rely heavily on envisioning (Walker, Winner, Hetland, Simmons, and Goldsmith, 2011). We collected data from 36 college undergraduates attending Winthrop University in North Carolina (USA): 18 studio art majors and 18 psychology majors. Students in both groups had completed an average of 6.5 semesters of college (they were mid-way through the first half of their final year), and participants in both groups had, on average, taken about the same number of mathematics classes (psychology majors averaged 1.7 years of math in college and 4 years in high school; studio arts majors averaged 1.4 years of math in college and 3.8 years in high school). The arts majors had completed an average of 16 undergraduate art courses; the psychology majors had taken no more than one arts course while enrolled in the college.

We administered a Geometric Workout assessment (Callahan, 1992) and the two verbal intelligence scales of the Kaufman Brief Intelligence Test (KBIT). Groups of students completed the tasks individually during designated testing sessions apart from their regular class meetings. The geometric reasoning test was based on a series of tasks created by mathematician Patrick Callahan for his college-level mathematics classes. The 27-item test required students to rely on visual working memory as they engaged in various spatial transformations. Examples of problems from the test include the following:

Imagine you have two squares of the same size. You place one square on top of the other, rotating the top square 45 degrees. (Remember that 45 degrees is half of 90 degrees.) What shape is the overlapping region? Try to figure out the answer in your head without drawing. Describe your answer in words as best you can. (Answer: an octagon or other 8-sided figure)

Below are pictures of "nets." You can fold them on the solid lines to make 3-dimensional forms. Circle the one that can be folded into a closed form (that is, one that has no holes or openings). (Answer: Net 1)



Net 1 *Net 2*

The verbal tasks included naming pictures and using a clue to complete a word with missing letters (e.g., BR_N: "a dark colour" [brown]; _ _ RE_ _ I _ I _ Y: "due to chance or fate" [serendipity]).

Studio arts students' performance on the geometry test was, on average, significantly higher than that of the psychology group (Figure 1). Regression analysis indicated that, when controlling for the effects of verbal intelligence (as measured by the KBIT), training in the arts was a statistically significant predictor of performance on the Geometric Workouts task (b=.34, t(34),=2.31, p=.027).

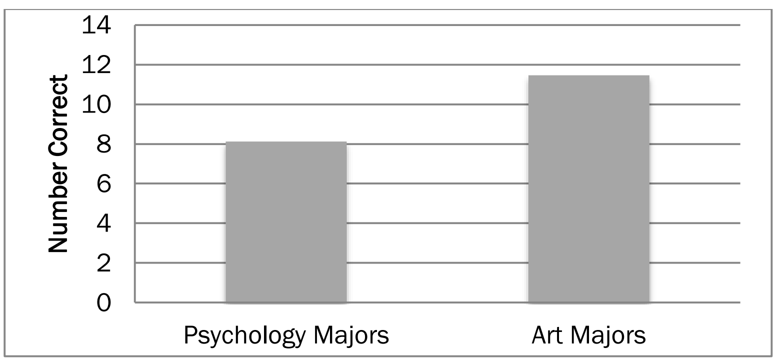


FIGURE 1: GEOMETRY WORKOUT SCORES: GEOMETRY AND STUDIO ARTS MAJORS

Study 2: Exploration of Students in Three Different Drawing Classes

As a follow-up of this study, Simmons and his colleagues conducted a pilot exploration of the effect of college level drawing classes on students' abilities to engage in artistic envisioning. One of the goals of this work was to understand whether art students do, in fact, develop geometric envisioning skills through taking art/design classes, or whether they come to art programs because they already have such innate and/or previously trained capacities. More immediately, the researchers, as faculty members in Fine Arts and Visual Communication Design programs, wanted to see whether such skills were currently being taught in their departments' foundation drawing classes.

Simmons and colleagues collected drawings from 58 first and second year college undergraduates who were enrolled in three different semester-long drawing courses: first semester design drawing (16 Visual Communication Design majors), first semester fine arts drawing (32 Fine Arts and Art Education majors), and Illustration I (10 Illustration majors). Of these three groups, students in the design drawing class were on average the least experienced in drawing, while the students in the illustration class were the most advanced, having already taken foundation design drawing classes. Students in all three groups completed three drawing tasks at the beginning and end of the semester; each task involved envisioning (visualising and mentally manipulating shape and spatial relations). These tasks are briefly described below.

Descriptive geometry drawings (Watts and Rule, 1946). These drawings required the translation of three orthographic views of a cuboid object (plan view, front elevation and side elevation) into a drawing as an isometric projection (Figure 2). The task began with rather simple problems that became increasingly complex; students were to draw as many as they could in the 10 minutes allowed. This kind of exercise can be used to develop designers' capacities to imagine an object as a solid form that they could then imaginatively rotate to view from any angle (Hanks and Belliston, 2008).

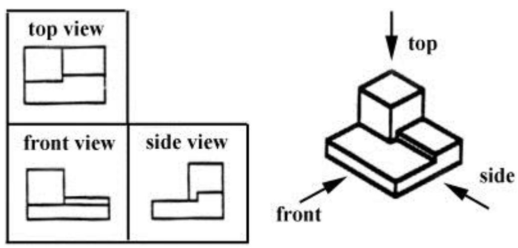


FIGURE 2: ORTHOGRAPHIC DRAWING OF A SIMPLE BUILDING (BLANCO, WILSON, JOHNSON, AND FLEMINGS, N.D.)

Drawing objects in perspective. This task involved using an orthographic drawing of a set of simple objects, shown in plan and front and side elevations, to draw the arrangement of objects in perspective as might be seen from a given viewpoint (indicated by an arrow). Successful drawings represented the envisioned objects with proper positions and proportions as well as with accurate overlaps as one form would have appeared when seen behind another (Figure 3).

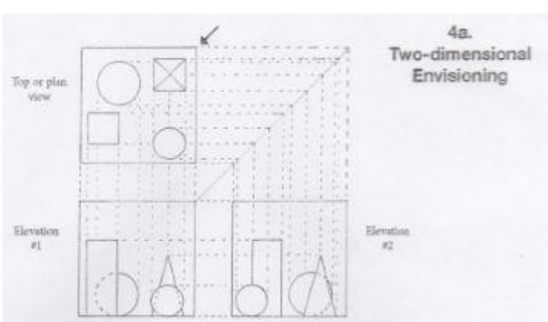


FIGURE 3: DRAWING OBJECTS IN PERSPECTIVE FROM A PLAN DRAWING (COURTESY OF D. G. BROWN)

Drawing a plan view from a perspective view. This task was essentially the opposite exercise; students started with a perspective drawing of the same objects and were to delineate the arrangement in a plan view (Figure 4).

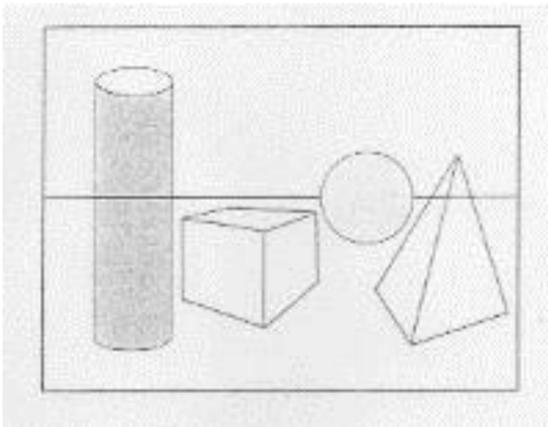


FIGURE 4: DRAWING A PLAN VIEW FROM A PERSPECTIVE DRAWING (COURTESY OF D. G. BROWN)

We did not conduct statistical comparisons of the drawings in the three groups, but holistic comparisons of the pre- and post-course drawings suggested that, students in fine arts and design drawing classes showed little to no change in their abilities to perform these three types of drawing tasks; students in the illustration class did evidence some global improvement on all three. Some of the students, especially those in the fine arts class, failed to see a connection between these tasks and their ideas about what constituted drawing. A number of students voiced resentment at having to complete the envisioning tasks, particularly the descriptive geometry problems, and some found the task virtually impossible to understand even after several demonstrations by their teachers. We wondered whether their resistance, sometimes expressed rather vehemently, simply reflected students' lack of experience doing such tasks or perhaps a deep-seated prejudice against having to use logic and mathematical skills in the service of art. We will return to these questions and to other implications of this exploration with drawing classes in our conclusion.

Study 3: Longitudinal Study of Geometric Reasoning in High School Students

The third study, still in the data analysis phase, is a Boston-based, quasi-experimental longitudinal investigation of the same general question: Does study of visual arts help students develop envisioning skills that support geometric reasoning? We are comparing three groups of 9th graders on measures of spatial geometric reasoning and artistic envisioning as well as on standard measures of spatial skill: visual arts students (VA), theater students (T) and students taking intensive squash lessons (SB).

The visual arts participants (VA) attended an arts-based public high school, where they spent the mornings studying core academic subjects and the afternoon studying visual arts. The 9th grade visual arts curriculum includes a year-long drawing class. A small number of VA students were recruited from Artists for Humanity (AFH), an after-school, apprentice-style entrepreneurial arts program which students attend for nine hours each week. The theater students (T) attended the same arts-based high school and followed the same schedule as the VA students, studying drama and stagecraft in the afternoons. Students in the sports condition (SB) participate in an intensive, nine-hour per week after-school squash program. Virtually all students were from low SES urban populations and attended schools in the Boston public system.

We tested students at three time-points: beginning of 9th grade (pretest), end of 9th grade (posttest 1), and end of 10th grade (posttest 2). At each testing point, students completed the measures over two sessions approximately one week apart. Each session lasted approximately 70 minutes. The measures we used are as follows.

1. *The Geometry Reasoning Test was composed primarily of release items from NAEP, TIMSS, and PISA. Items were selected that tested reasoning rather than technical vocabulary or skill with formal proofs. The test has two equivalent forms administered in counterbalanced order at pretest, posttest1 and posttest2.*
2. *The Art Envisioning Test was developed for this project. It assesses aspects of artistic envisioning that have spatial qualities:*
    - *translating between 3-D and 2-D (drawing a 3-D scene from life; re-creating a pictured scene with 3-D figures)*
    - *abstraction (rendering a complex figure in terms of underlying simplified forms)*
    - *projection (drawing and/or identifying shape, size, and direction of cast shadows given an imagined light source)*
    - *mental rotation (drawing a scene from life from a 180 degree change of perspective)*
3. *Spatial Factors tests that assess the three major "pure" spatial factors (Linn and Petersen, 1985)*
    - *spatial perception (water levels task, Liben, 1991)*
    - *mental rotation (Vandenberg and Kuse mental rotations; Peters et al., 1995)*
    - *spatial visualisation (paper folding; Ekstrom, French, Harman, with Derman, 1976).*
4. *Control tests that assess abilities for which we hypothesised no differences among the three groups*
    - *CogAt measures of verbal abilities (Lohman, 2002)*
    - *Empathy (Joliffe and Farrington, 2006)*

We hypothesised that the VA students would outperform the T and SB students on all but the control measures of artistic envisioning (i.e., the Geometric Reasoning, Art Envisioning, and Spatial Factors tasks).

Due to the limited class sizes at the arts school, we collected data in three waves over a three year period. Data analysis has been completed to date for the first two waves of data—a sample of 108 cases (41 VA, 35 T, and 32 SB students). Results to date indicate that students in the VA group had statistically higher mean scores on both pre- and post test 1 geometry than did the other two (control) groups ($p < .05$; see Figure 2).

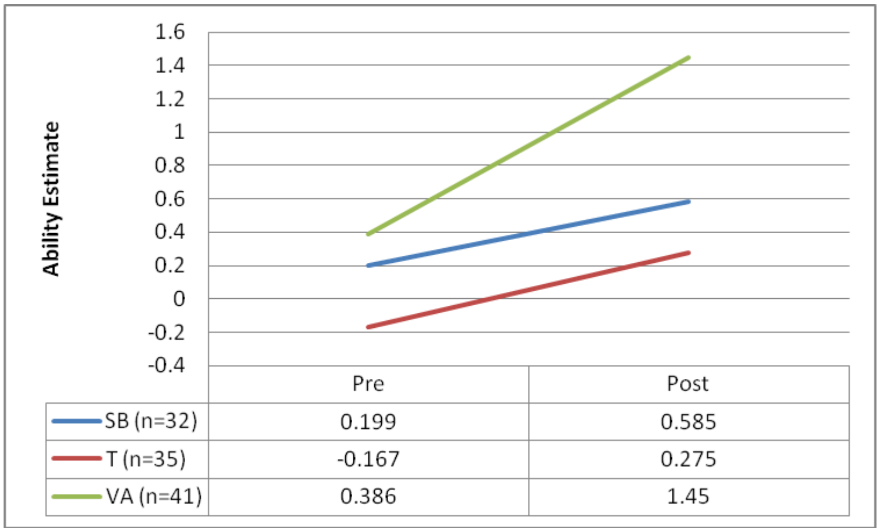


FIGURE 2: PRE AND POST-TEST SCORES ON GEOMETRIC REASONING TEST: VISUAL ARTS (VA), THEATER (T), AND SQUASH (SB) STUDENTS

Controlling for pretest differences, membership in the VA group remained a significant predictor of post-test1 geometry scores ($p < .05$; see Figure 2). Additionally, scores on the artistic envisioning pretest were a significant predictor of posttest 1 geometry scores ($p < .05$). At baseline, the data analysed to date indicate that the VA group also outperformed the other two groups on the standardised tests of spatial reasoning, but not on the two control tasks, as we had hypothesised.

Despite these findings suggesting that levels of visual arts thinking are linked to levels of geometric reasoning, we were not able to demonstrate that visual arts training, when compared to the theater and sports training, fostered greater *improvement* on the artistic envisioning test. Thus our hypothesis that more art instruction would cause higher levels of artistic envisioning and predict gains in geometric reasoning has not been borne out. Further work would be needed to determine whether the lack of selective improvement by the arts group on the art envisioning test reflects limitations in the “construct validity” of the art envisioning test; that is, that our art envision test did not assess the kind of envisioning that led to the art group improving more than the control group on geometric reasoning.

The analysis of two of our three waves of data indicates that VA students performed better than the other groups on the tasks tapping visualisation skills. However, given that the students self-selected into visual arts, theater, and sports, we cannot determine whether the observed difference is a result of spatially skilled students self-selecting into the visual arts, or whether study of the visual arts promotes the development of envisioning skills that are applicable to other domains of study, or both. A definitive answer requires conducting an intervention study which randomly assigns students to drawing and non-drawing "treatments."

In the next section, we conclude with some implications for drawing research and drawing instruction, considering how we might intentionally strengthen art students' capacities for geometric reasoning and related envisioning abilities. For even if it turns out that art students come innately equipped with capacities to reason geometrically, these capacities warrant intentional cultivation if they are to become active aspects of their creative problem-solving tool kit, especially if such problem-solving is eventually to be applied across disciplines in later studies and work.

IMPLICATIONS: ENVISIONING THROUGH DRAWING, IN ART AND ACROSS DISCIPLINES

The way in which we distinguish among disciplines and divide up the curriculum positions our educational system poorly for taking advantage of the potential for cross-discipline synergies such as those we explored in our studies of artistic envisioning. Yet the separation of "academic" and "artistic" disciplines has not always been the case. Art academies commonly included rigorous education in mathematics. From their beginnings in the 15th century under the inspiration of multi-disciplinary figures such as Leonardo da Vinci (McMahon, 1956) to their demise in the mid-twentieth century under the attack of Modernism, art academies commonly included rigorous education in mathematics (Pevsner, 1973). Moreover, even for those outside the arts, drawing courses involving explicit mathematical components were, until relatively recently, rather commonplace in public education throughout Europe and the United States (Efland, 1990; Stankiewicz, 2001).

The potential dividends of a more inclusive education can be considerable. Consider the histories of scientific innovators such as Kekulé, who attributed his capacity to visualise in chemistry to his early training in architecture (Rocke, 2010), and Nobel laureate physicist Luis Alvarez, who attended an "arts and craft" high school where the curriculum included industrial drawing and woodworking. Alvarez credited these experiences as contributing to his ability to both visualise the phenomena he studied professionally and also to design and build his own experimental equipment (Alvarez, 1987; Root-Bernstein and Root-Bernstein, 2013). Innovator and entrepreneur Steve Jobs also credited the arts, in particular a calligraphy course in college, as an important factor in his accomplishments at Apple (Isaccson, 2011).

Clearly, the needs of the future must be met by people who can cross boundaries and synthesise knowledge from disparate domains (Gardner, 2008), bringing diverse ways of problem posing and problem solving to bear on the pressing concerns we face today and on the unforeseeable challenges we will face in years to come. The process of developing such boundary-crossing minds in larger number may begin with conversations across professional cultures like those happening at conferences on drawing and cognition. Next steps must involve more concerted efforts at both research and educational interventions. In regard to the former, the projects addressed in this paper, and other studies recounted in this volume, should provoke more expansive and extensive research into cognition in and through drawing.

Other projects might include a survey of college-level foundation drawing classes and high school art classes to determine how drawing is currently taught and to gather information about the percentage of class time devoted to drawing that trains the kinds of artistic envisioning/reasoning that we have suggested might be transferrable to STEM domains. A related question is: to what degree do design schools still teach traditional skills like descriptive geometry and linear perspective vs. relying on digital design programs like *Revit*?

This kind of survey approach could lead to inquiry into the benefits of drawing by hand vs. using digital media, whether in design, illustration, or in fine arts. Concerns about such issues, and the larger question of what capacities (perceptual and conceptual) might be lost if designers and architects stopped drawing altogether, prompted a conference at the Yale School of Architecture in 2012 entitled *Is Drawing Dead?* (Yale, 2012). The answer was “not yet,” with many participants insisting that hand drawing is still essential for ideation despite admitting the value of digital media for technical drawing and rendering.

Another line of work would involve developing greater understanding of existing connections between STEM studies and drawing as a way of providing models and design principles for future programs. With funding from the National Art Education Foundation, Andrea Kantrowitz and Seymour Simmons are gathering exemplars of K–12 teachers’ uses of drawing to connect art with academic disciplines including, but not limited to, STEM studies. Should such research bear fruit, further steps may involve developing and implementing teacher-training programs that will prepare art educators, classroom teachers, and non-art specialists to teach drawing and integrate it throughout the curriculum.

Programs like these would, however, likely challenge several of contemporary education’s self-imposed segregations—not only the general boundaries between art and academics, but also those within visual arts education itself, such as the ever-growing divide between fine arts and illustration, and between fine arts and design. Divisions occur even within and among drawing programs themselves, including conflicts between those who promote academic drawing and those who favour more contemporary approaches, between those who emphasise drawing from observation and those who prefer drawing from imagination, or who focus solely on conceptual or abstract drawing.

While valuing each of these approaches and celebrating the differences among them, it is equally useful to identify fundamental continuities uniting them all, continuities with implications for learning and creativity within the wide spectrum of visual arts and across the disciplines. Additional factors that might be considered in a more inclusive view of drawing instruction include the following:

1. *Drawing study may support deeper development of artistic (and STEM) envisioning if it addresses a range of problem-solving challenges and incorporates a range of problem-solving strategies such as the use of descriptive geometry and other visualising practices.*
2. *Drawing instructors could make explicit the connections across disciplines, both in terms of input and output. This kind of "teaching for transfer" is what Perkins and Salomon (1988) call "the high road to transfer" (Perkins and Salomon: 25-28). In terms of input, for lessons that "draw upon" knowledge from other domains like natural science, logic, or mathematics, cross-disciplinary connections would need to be brought directly to students' attention, encouraging them for example to recall what they learned in drawing class and use the knowledge and skills, as appropriate, in geometry class. In short, where drawing skills learned in the context of art have potential applications across disciplinary boundaries, these should be made evident through examples, as well as suggestions about possible future applications.*
3. *An implication of the search for productive "conversations" between the arts and STEM work is to consider creating classes that are co-taught by teams of professionals from art and non-art disciplines in which common principles and strategies are discussed and explored. These collaborations may also create productive environments for generating and testing theories to advance cross-disciplinary thinking, visualising, and problem solving through drawing.*

We are admittedly not the first to make suggestions like those above, and many of our readers may already be involved in such enterprises. An example of what we call for can be seen, for example, in Josef Albers' innovative, comprehensive drawing courses at the Bauhaus, Black Mountain College, and Yale University, which incorporated a range of non-representational and representational drawing skills applicable equally to design and fine arts (Horowitz and Danilowitz, 2006). Another example can be seen in the comprehensive drawing program at Pratt Institute in New York requiring all first-year students to study six distinct but interrelated drawing strategies, including gestural drawing, mark making, and geometric construction (Fasolino, Wirls, and Sloan, 2008). Pratt recently demonstrated its commitment to building cross-disciplinary connections by establishing an endowed chair for mathematics and the arts (Pratt, 2012).

At the K–12 level, several examples of drawing as a cross-curricular vehicle for learning and creating can be found in Ron Berger's 2003 volume, *An ethic of excellence: Building a culture of craftsmanship with students.* Grounded in Berger's background as a carpenter and his expertise in "Expeditionary Learning," his 6th grade curriculum emphasised experiential engagement, individual initiative, team-work, craftsmanship, and cross-curricular problem-solving. The Reggio Emilia schools demonstrate that even very young children can benefit from integrating drawing with "academic" subjects, where drawing is one of their most important "languages of learning" (Edwards, Gandini, and Forman, 1998; Giudici, Rinaldi, and Krechevsky, 2001).

The research described in this essay, taken together with similar topics addressed throughout this edition of TRACY, suggests the potential for a paradigm shift to a more central role in education given to drawing, in its myriad forms, as a means of visualising, representing, and communicating complex ideas. With greater recognition of the importance of "graphicacy" (thinking in images) as a complement to numeracy and literacy at the core of the curriculum (Garner, 2010), it may be possible to help the broader culture re-envision a central role for the arts in education—as a means of connecting, rather than separating, domains of skill and knowledge, and of understanding learning as both "hands on" and "minds on" problem-solving. It is up to those who study drawing, teach drawing, and prepare future drawing teachers to help insure that these potentials are actually realised.

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