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More Axopictures from Axoplans

Physical and Pure Geometry

When infants are given coloured shapes to play with they begin to study physical geometry. The toys have various defects of materials and workmanship. Some students play with imaginary perfect toys which have no defects. That is pure geometry.

In schools, Physical Geometry is dealt with in the Art and Craft Departments. Pure Geometry is the province of the Mathematics Department. The two kinds of geometry are not allowed to mix.

Engineers and some other applied scientists and technologists often combine pure and physical geometry and thereby create great inventions and magnificent works. They have to be careful that they do not also create confusion in their own and other people's imaginations.

Drawing

Drawings are two dimensional models of three dimensional shapes. They are part of physical geometry.

No drawing can have all the qualities of the original object. Drawings may be made in various ways each retaining some of the qualities of the object and losing others. We choose Axometric Drawing because it has the quality of mobility which it gains at only slight cost in visual similarity (the loss of converging lines and vanishing points).

In axometric drawing the imaginary lines through object points and their corresponding picture points are at right angles to the flat picture surface.

We found that axometric drawing offered many opportunities for development which had been somewhat neglected.

We often use the word *Sketch* in place of drawing. In the Axoplan Papers a sketch is drawn without instruments and follows a definite geometrical shape but a greater degree of roughness is permitted than in precision drawing as is necessary when sketches are made under somewhat rough field or site conditions by hand.

There are various entrances to the study of Geometry. At a short course on Measured Pictorial Sketching for Teachers held at Bristol University in May 1979 some teachers expressed a preference for an introduction to the Axoplan Method through the circle and ellipse. This paper has been written following that meeting.

The Turns

The geometry of rotation about a fixed axis can be observed by picking up a twig with two branches and twirling it between the fingers. The twig rotates about a line (the axis) through its ends and the ends of its branches move in circles which are at right angles to the axis, therefore parallel to one another. Perhaps this was one of the early observations of the physical geometry of Nature.

This is the Geometry of the Potter's Wheel and and of the Centre Lathe which were both named *The Turns* about 2000 years ago.

In this paper we merely state the rules for converting a set of circles in an object into its axometric picture on a flat surface.

We imagine a horizontal ground plane, a vertical picture plane and an object plane which might be parallel to the picture plane (PP.)

If the object plane is parallel to the picture plane a circle on it is the same size in the picture as in the object.

Figure 1

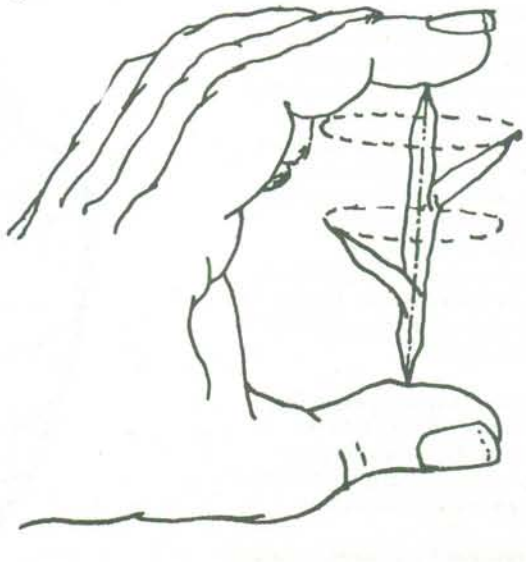
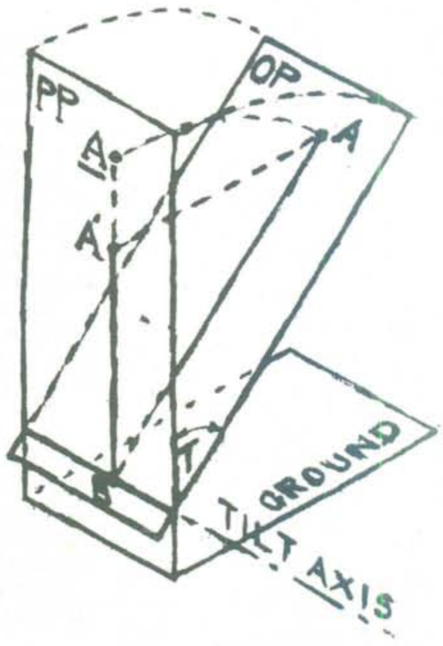


Figure 2



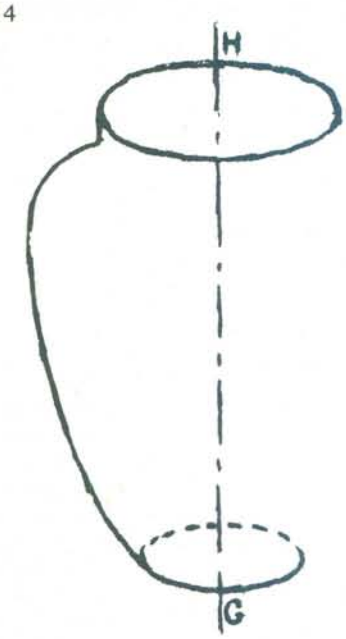
If the object plane is not parallel to the picture plane it cuts the picture plane in a line which we name the tilt axis.

A circle on the object plane is seen in the picture as an ellipse with its major axis parallel to the tilt axis. The minor axis is equal to the major axis reduced by the tilt ratio which is a constant depending on the angle of tilt of the object plane. All circles on the object plane or on any plane parallel to it are converted into ellipses of the same proportions.

If the centres of circles in a three dimensional object lie on a straight line the centres of the ellipses that represent them in an axometric picture also lie on a straight line.

These facts make it economically practicable to provide sets of ellipses of the same proportions to guide the axometric sketching.

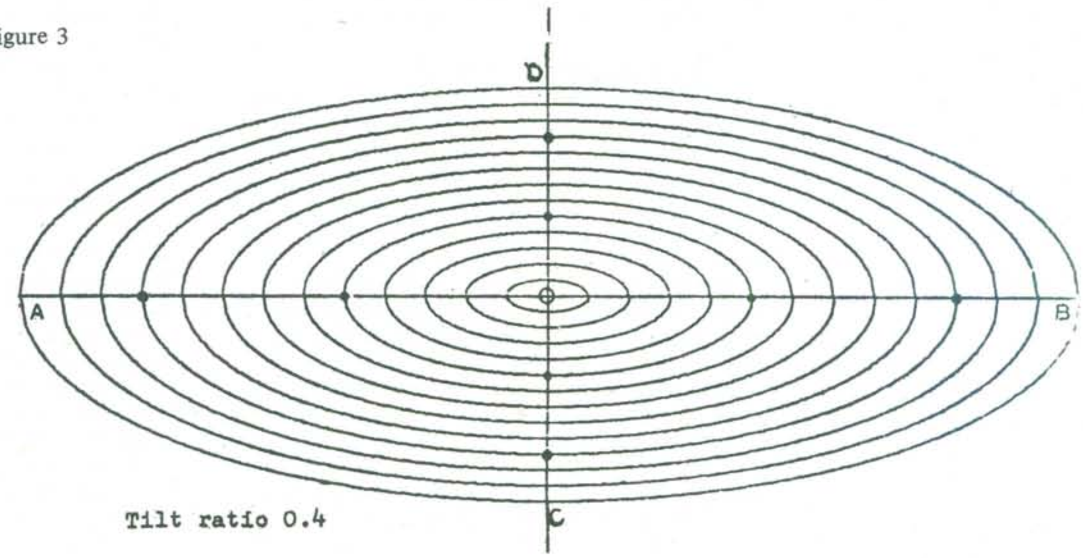
Figure 4



The choice of the ellipses and the curve which connects them is with the Sketcher.

Place ellipses from the set to touch, but not cut, the outline curve. The line GH always coinciding with axis EF. If there are not enough ellipses in the set sketch some more between them. (Interpolation).

Figure 3



Ellipses are troublesome and costly to draw well so the work of producing an original set of ellipses from which drawings may be traced is given to a computer controlled drawing table. An example of a set of ellipses of tilt ratio 0.4 is given above.

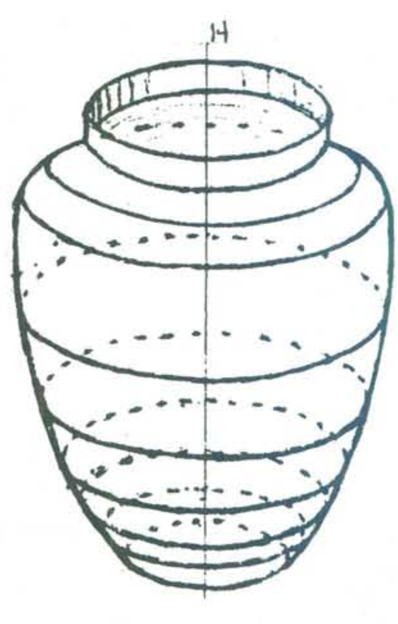
In the object the axis AB and axis CD through the origin O are in a plane at right angles to the axis EF of the Wheel or Lathe. In the picture this axis coincides with the minor CD axis and is extended beyond it, line EF in the figure.

This set of ellipses can be used immediately to illustrate some products of the Potter or Turner or to give a uniformly divided scale in any direction from O in the plane of the ellipses.

On a sheet of tracing paper draw a centre line GH. Also trace from the set of ellipses one to represent the top of a vase and one to represent the bottom – this may be traced lightly in dashed line because part of it might be hidden by the finished vase.

Connect the top and bottom by a curve to satisfy the eye to represent the appearance of the vase.

Figure 5



This method of sketching a picture of a vase is remarkably like the process of throwing a pot on a wheel. The Potter has a shape in imagination. The wheel ensures that the shape consists of circles at right angles to a line axis on which their centres lie. The axoplan method responds to the Artist's imagination of a shape but ensures that the picture is composed of pictures of circles at right angles to the axis on which their centres lie.

Notice that measurement does not enter into the process and that by using different sets of ellipses vases of the same shape may be viewed from different angles.

Students who have been shown how to manipulate the clay on a potter's wheel may be left to experiment with it, guided by their imaginations and physical reactions. Their freedom of action is combined with the discipline of the wheel. Similarly, students who have been given a set of ellipses and shown how to assemble them on a single line axis may be left to experiment with them without further instruction.

The shape of the outline of a vase or dome depends on the direction from which it is viewed. Sometimes a more precise description of the shape is desired. This may be provided by the section of the vase when it is cut by a plane through the axis.

When a picture of the section of a vase has been given, the picture of the vase may be assembled by placing the major axes of the ellipses so that their ends just, and only just, touch the section curve.

The ellipses are traced one by one and the enveloping curve drawn.

The shape of the section may be given simply by drawing it or it may be expressed as numerical co-ordinates which can then be mapped on to a graph paper or produced in a machine by numerical control.

The circle is a powerful figure in plane geometry and may be used in a great variety of ways. Some of the ways may also be applied to the ellipse. When a process is applicable to circles on a plane and ellipses which are axometric projections of the circles it is said to be invariant under axometric projection.

If in a circle a diameter LM is drawn through the mid point F of a chord DE, the diameter and chord are at right angles. This is invariant on axometric projection in the sense that in Figure 8a DEJH is any circle with centre C and DE any chord. F is the mid point of DE. LM is a diameter through F. Figure 8b is an axometric picture of Figure 8a.

Figure 6

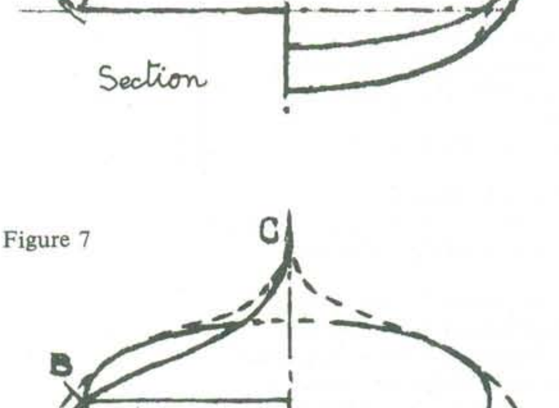
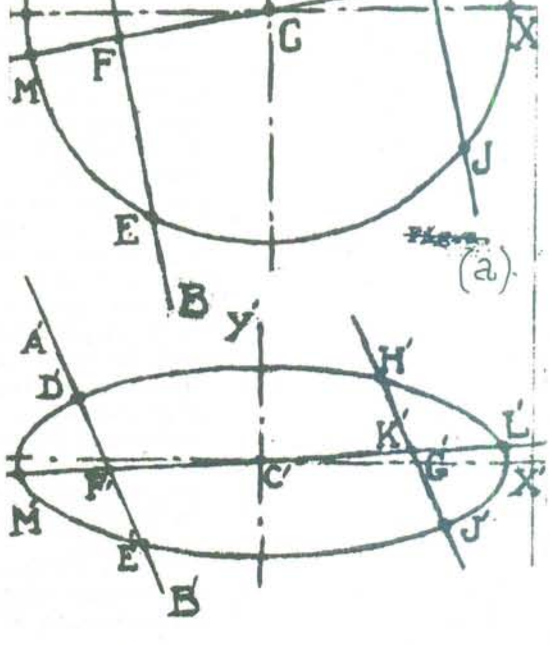


Figure 7

Figure 8 (a)
and Figure 8 (b)



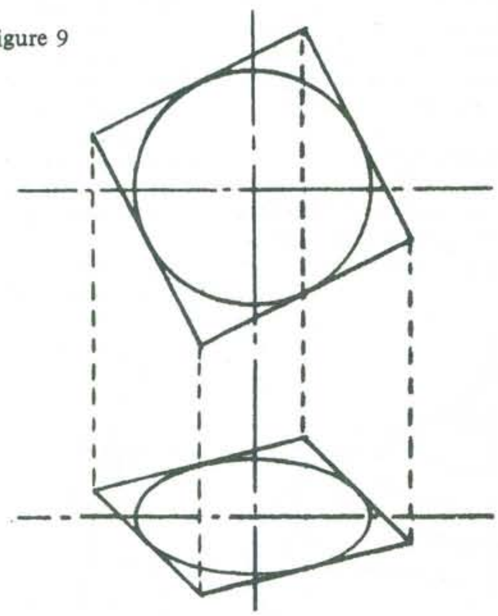
The picture of the diameter is 'L'M', F' is the mid point and the angle D'F'C' is a picture of a right angle.

This construction has the advantage that any chord HJ parallel to DE has a mid point K on the line F'C'. Thus it offers accuracy and a check.

When two lines at right angles have been found two parallel tangents may be drawn to each line to form a tangent square to the circle. The construction may also be applied to the ellipse which is a picture of the circle.

Sometimes it is convenient to draw a square by drawing two pairs of parallel tangents to a circle at right angles to one another. The method is directly applicable to an ellipse.

Figure 9

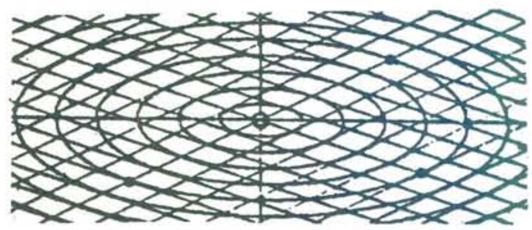


We now have three methods for drawing an axometric picture of a circle inscribed in a square.
(i) The cumbersome projection from two other views (plan and elevation)
(ii) By reduction of lines at right angles to the tilt axis and
(iii) the direct construction shown above.

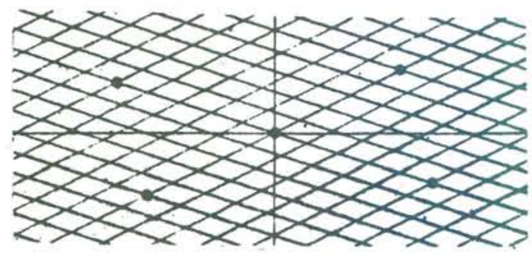
The tendency is to develop the internal geometry of the axometric picture because it is compact, direct and quick.

Notice that if the circle is turned about its centre the drawing is unchanged because every point on the circle moves into a position previously occupied by another point on the same circle. This is also true of the axometric picture which is unchanged as the circle turns. The temptation to turn the ellipse as a whole must be resisted. But when the tangent square is turned about its centre it occupies a new position (unless it is turned a whole number of right angles).

If a set of ellipses (an axometric picture of a set of concentric circles with equal increments of radius) has been drawn, an axometric picture of a square chequer board may be made by drawing two orthogonal sets of tangents to the ellipses on a tracing placed over them. When the ellipses and the tangents are printed together a combined space-guide is produced.



This is useful for some purposes but when the tracing is printed separately the simple chequer board is produced and is preferred for its simplicity for many purposes.



The first sets of ellipses used in Operation Axoplan were produced by Ian T. Parsons at the Computer Unit of Bristol University.